Probing the interior of the Schwarzschild black hole

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May 4, 2022

**Abstract**

The probing of the black hole interior is studied beginning with the black hole metric [1] where the shear parameter and the vorticity parameter can be computed. From the following set of parameters, the rate of change also known as the Raychaudhuri equation are able to be determined. It is found that the vorticity computed is zero which is consistent with a black hole of zero spins. Classical equations of motion are also plotted as a function of .

**1 Spacetime Metric as a 4x4 matrix and its inverse**

The spacetime metric in this region is written as

where, N is the lapse function that determines the foliation and

The spacetime metric is then define as

The inverse becomes

Multiplying them together gives us

**2 Christoffel Symbols**

Christoffel symbols of the second kind as a function of the metric tensor is defined as

where is the inverse of the matrix

The non -zero terms of the computed Christoffel symbols are

**3 Components of the tangent vectors to radial timelike or null geodesics**

Rearranging our metric from Eq.1 gives

Consider timelike, radial geodesics:

This gives us

Factorizing the right- hand side of the equation

Given that , we know from the above equation that

Solving for

Then the tangent vectors to radial timelike geodesic is

**3.1 Choosing suitable to simplify components**

Simplifying our equation, we choose a suitable choice for as

to obtain

**4 Transverse metric**

The transverse metric is defined by

where

Using this form of vector from Eq.14, we can easily obtain the transverse metric as

**5 Covariant Derivative**

The covariant derivate is defined by

which becomes

**6 Expansion scalar**

Now, the expansion scalar is defined as

Using Eq.18 and metric from Eq.3, the expansion scalar becomes

**7 Shear Tensor**

For shear tensor, it is given by

where .

Shear is defined by

From Eq.22, is identifies as

By solving the above equation, the shear obtain is

**8 Vorticity or Rotation Tensor**

Vorticity is given by

whereby

Vorticity is defined by

Similarly, from Eq.25, is identified as

Since is a null matrix, is therefore

**9 Raychaudhuri Equation**

The behavior of geodesics on how a congruence of timelike or null geodesic evolves over time is one way to probe the classical and effective structure of spacetime. This analysis is closely linked to the geodesic deviation and the expansion scalar and its rate of change that is the Raychaudhuri equation where it is found to be

where is the shear parameter, is the vorticity parameter and is the proper time along the geodesic.

The Raychaudhuri equation is then computed which yields

**10 Plotting the Classical Equation**

The following phase space trajectories obtained from the classical Hamiltonian is

whereby the range is where corresponds to the black hole’s event horizon and corresponds to the classical singularity.

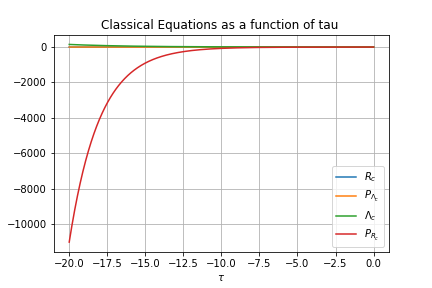


Figure 1: Plotted graph of classical equations as a function of tau

To solve them numerically, more research is required but nevertheless the quantum equation should converge with the classical equations at the event horizon [1].

**References**

[1] E.Alesci, S.Bahrami, D.Pranzetti (2020), *Assymptotically de Sitter universe inside a Schwarzschild black hole,* Phys.Rev.D 102, 066010

[2] S.Rastgoo (unpublished paper)

[3] S.M. Caroll, (June,2003), *An Introduction to General Relativity Spacetime and Geometry*,